

$$\frac{\cos y + \cot y}{1 + \sin y} = \cot y$$

$$= \frac{\cos y + \frac{\cos y}{\sin y}}{1 + \sin y} \cdot \frac{\sin y}{\sin y}$$

$$= \frac{\cos y \sin y + \cos y}{(1 + \sin y) \sin y}$$

$$= \frac{\cos y (\cancel{\sin y} + 1)}{(\cancel{1 + \sin y}) \sin y}$$

$$= \frac{\cos y}{\sin y}$$

$$= \cot y \quad \boxed{\text{QED}}$$

$$\sec^4 \theta - \tan^2 \theta = \sec^2 \theta + \tan^4 \theta$$

$$\hookrightarrow = (\tan^2 \theta + 1)^2 - \tan^2 \theta$$

$$= \tan^4 \theta + 2\tan^2 \theta + 1 - \tan^2 \theta$$

$$= \tan^4 \theta + \tan^2 \theta + 1$$

$$= \tan^4 \theta + \sec^2 \theta$$

QED

Prove $\frac{1}{1 - \tan x} + \frac{\tan x}{1 - \cot x} = 1 + \tan x$.

$$\hookrightarrow = \frac{1}{1 - \tan x} + \frac{\tan x}{1 - \frac{1}{\tan x}} \cdot \frac{\tan x}{\tan x}$$

$$= \frac{1}{1 - \tan x} + \frac{\tan^2 x}{\tan x - 1}$$

$$= \frac{1}{1 - \tan x} - \frac{\tan^2 x}{1 - \tan x}$$

$$= \frac{1 - \tan^2 x}{1 - \tan x}$$

$$= \frac{(1 - \cancel{\tan x})(1 + \tan x)}{1 - \cancel{\tan x}}$$

$$= 1 + \tan x \quad \boxed{\text{QED}}$$

$$\frac{\cot^2 \beta - \csc^2 \beta}{1 - \sin^2 \beta}$$

$$= \frac{-1}{\cos^2 \beta}$$

$$= -\sec^2 \beta$$

$$\frac{\csc t + \sin(-t)}{\cos(-t) - \sec t}$$

$$\cos(-t) - \sec t$$

$$= \frac{\csc t - \sin t}{\cos t - \sec t}$$

$$= \frac{\frac{1}{\sin t} - \sin t}{\cos t - \frac{1}{\cos t}} \cdot \frac{\sin t \cos t}{\sin t \cos t}$$

$$= \frac{\cos t - \sin^2 t \cos t}{\sin t \cos^2 t - \sin t}$$

$$= \frac{\cos t (1 - \sin^2 t)}{\sin t (\cos^2 t - 1)}$$

$$= \frac{\cos t \cos^2 t}{\sin t (-\sin^2 t)}$$

$$= -\frac{\cos^3 t}{\sin^3 t} = -\cot^3 t$$